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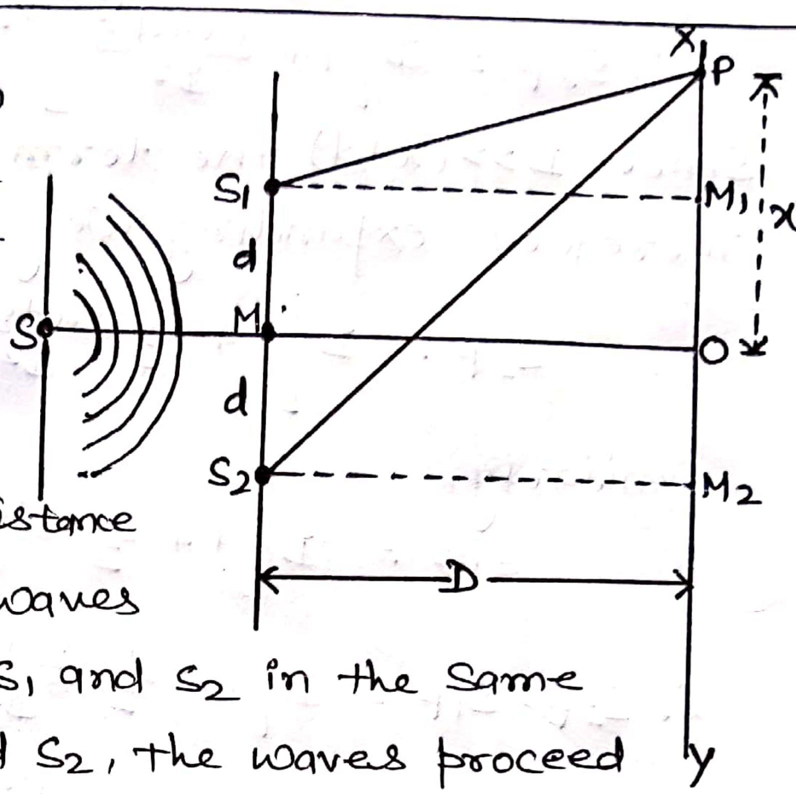
# Deduction of formula $\lambda = \frac{2xd}{D}$




Let S be a narrow slit illuminated by monochromatic light and $S_1$ and $S_2$ are two parallel narrow slit very closed together and equidistance from S. The light waves from S arrive at $S_1$ and $S_2$ in the same phase. Beyond $S_1$ and $S_2$, the waves proceed as if they started from $S_1$ and $S_2$ produced an interference pattern on a screen xy placed parallel to $S_1$ and $S_2$.

Let us join $S_1$ and $S_2$ and bisect the line $S_1S_2$ at M. From M, let us draw a perpendicular MO on the screen. Let P be any point on the screen and let

$$S_1S_2 = 2d, \quad MO = D \text{ and } OP = x$$

To find the intensity at P, join $S_1P$ and $S_2P$. The two waves are at P, having transversed different wave paths $S_1P$ and $S_2P$. Let $S_1M_1$ and $S_2M_2$ be perpendiculars on the screen.

Now

$$(S_2P)^2 = (S_2M_2)^2 + (PM_2)^2$$

$$= D^2 + (x+d)^2$$

$$= D^2\left[1 + \frac{(x+d)^2}{D^2}\right]$$

$$\therefore S_2P = D\left[1 + \frac{(x+d)^2}{D^2}\right]^{1/2}$$

Since $D >> (x+d)$ the term $\frac{(x+d)^2}{D^2}$ is very small

Therefore, expanding we get

$$S_2P = D\left[1 + \frac{1}{2}\frac{(x+d)^2}{D^2}\right]$$

Similarly,

$$S_1P = D\left[1 + \frac{1}{2}\frac{(x-d)^2}{D^2}\right]$$

$$\therefore S_2P - S_1P = D\left[1 + \frac{1}{2}\frac{(x+d)^2}{D^2}\right] - D\left[1 + \frac{1}{2}\frac{(x-d)^2}{D^2}\right]$$

$$= \cancel{D} + \frac{1}{2}\frac{(x+d)^2}{D} - \cancel{D} - \frac{1}{2}\frac{(x-d)^2}{D}$$

$$S_2P - S_1P = \frac{1}{2D}\left[(x+d)^2 - (x-d)^2\right]$$

$$= \frac{1}{2D}\left[x^2 + 2xd + d^2 - x^2 + 2xd - d^2\right]$$

$$= \frac{1}{2D} \times 4xd$$

$$\therefore S_2P - S_1P = \frac{2xd}{D}$$

Now, the intensity at point P is a maximum or minimum according as the path difference $S_2P - S_1P$ is an integral multiple of wavelength or an odd multiple of half-wavelength.

Thus for bright fringes (maxima)

$$S_2P - S_1P = \frac{2xd}{D} = n\lambda$$

$$\text{or, } x = \frac{D}{2d} n\lambda$$

and for dark fringes (minima)

$$S_2P - S_1P = \frac{2xd}{D} = (2n+1)\frac{\lambda}{2}$$

$$\therefore x = \frac{D}{2d}(2n+1)\frac{\lambda}{2}$$

Now let $x_n$ and $x_{n+1}$ denote the distance of $n$th and $(n+1)$th bright fringes. Then

$$x_n = \frac{D}{2d} n\lambda \quad \text{and} \quad x_{n+1} = \frac{D}{2d}(n+1)\lambda$$

$\therefore$ Spacing between $n$th and $(n+1)$th bright fringes is

$$x_{n+1} - x_n = \frac{D}{2d}(n+1)\lambda - \frac{D}{2d} n\lambda$$

$$= \frac{D}{2d}(n+1-n)\lambda$$

$$= \frac{D}{2d}\lambda$$

It is independent of $n$. Hence the spacing any two consecutive bright fringes is the same, it can shown that the spacing between two dark fringes is also $\frac{D}{2d}\lambda$.

The spacing between any two consecutive bright or dark fringes is called the 'fringes-width' and is indicated by $x$.

Thus $x = \frac{D}{2d}\lambda$ $\quad \therefore \boxed{\lambda = \frac{2xd}{D}}$